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Conditions for periodic and aperiodic behavior of formal neural nets *

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Abstract

Formal neural networks (FNN) can display dynamical behaviours, more or less different from each other depending on their units, the functions attributed to these units, interconnections, parameters, state spaces and initial states, etc. Whatever is 'chaos' – of which several practical and more exact definitions exist –, it used to be emerging at special conditions. Its prediction most often requires an individual analysis of the dynamical system (DS) in question. A study of such conditions is usually necessary in order to reach suitable control, which now seems to become a new trend in chaos theory. In chaos control tasks quick commands and at least short-term foresight of trends are required. It is a primary question also to define in advance what is regarded to be a controlled case of chaos. Possible importance of these general considerations at molecular scale is also discussed, avoiding not well-founded speculations.

Key words: Chaos; Diagnosis and therapy of chaos; Kinds of chaos control; Networks and chaos; Dechaotization; Small scale chaos

1. Introduction

The *'chaotic dynamics'* is an intensively investigated field but it is not easy to offer a unified definition [1–4]. Special aperiodicities, pseudorandomness and strong sensitivity to initial conditions, self-similarity are the most often mentioned items. Difficulties arise with respect of the notion of *'strange attractor'* as well, since for an attraction metrical space (i.e. a distance concept) and convergence should usually be strictly defined. These can be done here only through set/point distance where the set itself is *not* always clearly (i.e. a priori) defined. The fractional dimension of attractor set also occurs. The 'chaos' is a paradigm beside noisiness to interpret irregular nonlinear dynamics (NLD), in which the 'bad' noise and the 'good' dynamics cannot be separated as it is believed to be possible for noisy systems.

Irregular structures and behaviours make the chaos studies relevant for neurosciences. If disor-

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der is regarded as something bad, then the conditions of chaos are interesting in order to eliminate it. However, there is *no* general agreement when a disorder is to be regarded as 'good' or 'bad'. Often chaos represents a strictly undesired case when the control task is the dechaotization. The medical and technical disorders, both with noisy and chaotic dynamics are most often taken as something to be avoided and therefore the conditions of chaos are required to find also the ways of its elimination or damping it or its consequences. With the simplified terms of medical sciences – as paraphrases – the diagnosis of chaos may be necessary to offer a strict therapy. But it is still not necessarily sufficient. The therapy can be symptomatic too. The conditions and control of chaos are hardly separable issues since the control is achieved by changing conditions [5–8].

Here, the examples are mainly taken from the authors self-constructed neuron or neural net-like dynamical systems. In few cases other widely known examples will also be used to support statements.

2. Formal neural nets (FNN) and their diversity

Formal neural nets are neither only those of McCulloch and Pitts ([9] MCP) nor correspond alone to the new 'technology' – called artificial neural nets (ANN) – based on soft threshold operators (STO) and occasionally adaptive synthesis (learning or storage). Instead, practically any vector–vector map determining the behavior of a continuous or discrete dynamical systems (DS) can be regarded as a net (see e.g. [5]). It is best to define formal neural nets (FNN) as generally as possible and still remaining applicable. These are specified by state space, input/output signals, individual units and function attributed to them, a matrix enclosing either values or functions. This is still a time-discrete system of which the 'motion' is obtained by iteration. Continuous dynamics or even differential systems need special methods and considerations. Autonomous and forced systems are permitted equally, as well as their modifications.

3. Results

Three classifications at least are important for the systematic study of the conditions of chaos: (a) what kind of dynamical behaviours (DB) exist (e.g. chaos versus non-chaos); (b) what sorts of controls can be formulated; (c) what sorts of DSs exist?

Ad (a): convergent, divergent, chaotic, various families of oscillations, fixed states. This subject includes stabilities of which several ones can be defined as well as DSs [10].

Ad (b): small or short-lasting versus persistent or strong, control governing from chaos to chaos or to non-chaos; external or intrinsic (autonomous or not).

Ad (c): time or value discrete versus continuous, finite versus infinite, autonomous versus open, network or single variable system etc.

These aspects – whose very few examples are given here – define exponentially many fictituous situations of which some are realistic or have practical importance.

3.1. Are the chaotic activities predictable?

The chaos is per definitionem deterministic, thus its behaviour seems predictable but in practice the computation of all consecutive states between the initial state and the state to be predicted is hardly avoidable. Thus non-predictability is the usual opinion. Nevertheless, a 'statistical' prediction (at stationary conditions), e.g. amplitude spectra or in the case of neural nets inter-spike–interval spectra are more predictable (*spike is the well known name of pulse-like electrical neural signals*). Chaos in discrete systems may appear already at one dimension, in continuous DSs from dimension three. Nonlinearity is a necessary but not a sufficient condition [1–4].

3.2. In what sense the control of chaos is peculiar?

Recently Shinbrot et al. (6) expressed the view that small perturbations are good to control chaos and are not efficient to control other dynamical systems. Nevertheless, it can be demonstrated

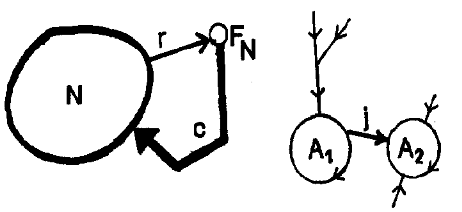

Fig. 1. Sensitive dependence on initial state of a MCP-FNN. Left: The network N is supplemented by a unit F_N recognizing its specific state. N consists of several interconnected units. The arrows r and c correspond to signals making recognition and commanding modification. Right: State transition graph with two originally disjoint A1 and A2 attractors with engulfing transients, while *j* means the possible state transition introduced by the F_N unit (section 3.2).

that this formulation of 'radical' differences in control principles is not tenable in general. For example the finite and discrete McCulloch-Pitts networks [5] are very sensitive to initial conditions with transition to the new attractor and are *not chaotic* at all, while small and short-lasting perturbations in chaotic systems may keep the DS *on the same chaotic attractor* in a statistical and metrical sense. Here, the opposite view is stressed: all achievements and methods or control principles are applicable in both chaotic and non-chaotic cases, the differences can be explored step by step in individual cases. This is now the state of the art (Fig. 1).

3.3. Cases of autonomous synchronous behaviour and regularity

This less chaotic chaos flow may occur in a system of many variables, when no external influences but instead intrinsic interactions are introduced in order to get regularization or some dechaotization. At weaker internal interactions, synchronization is obtained; at stronger ones, synchronous and highly regular component flows can be obtained in a net demonstrated in ref. [5]. These effects are persistent ones.

Even if the components of net are *not* interconnected at all, then a kind of short-lasting (transient) synchronous component motion can be achieved if very close but (pairwise) slightly different initial values are chosen. In such cases seemingly identical motions persist for an adjustable time. In chaotic systems, however, the measure of deviations accumulate, the initial deviations increase exponentially and a 'spontaneous' divergence from each other may take place. This is the *manifestation of the well-known butterfly effect* (i.e. sensitivity to initial value deviations) but the 'strange' attractor of identical components remains identical in each component (Fig. 2).

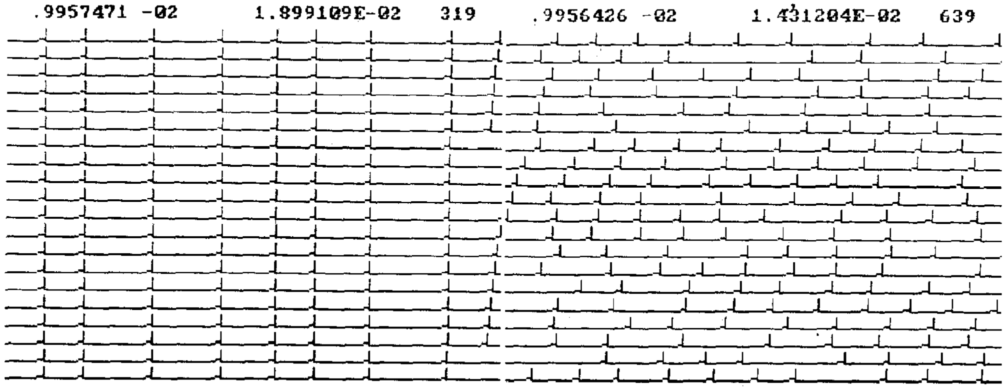

Fig. 2. Several PLMIs without interconnections. Their chaotic activity were initiated from almost the same initial values but with very small deviations. The activities remain simultaneous for a computable lifetime, after a spontaneous and essentially irreversible desynchronization. The component flows are different, their statistical properties remain identical (section 3.3).

3.4. Small but long-lasting (permanent) external control

A permanent – albeit small – control can evoke strong effects even in linear (thus non-chaotic) systems if the relaxation times are not enough to permit returning from perturbated state. This can be demonstrated already in first- or second-order linear iterations. Often the permanent effects accumulated rapidly but in linear systems a conventional convergence or divergence occur. The linear system attractors are not 'strange'. Example: $x_{n+1} = mx_n + b$ as easily computable. Dependence on m may lead to steady state or divergence. The last case occurs at abs$(m) > 1$ without perturbation.

3.5. Short, small and singular external control (pulse control)

Short and rarely applied control pulses might be ineffective in chaotic cases as well: the motion remains inside or near to the "strange" or "peculiar" attractor set but completely different motions occur. In a DS with a strange attractor the similarity of the motions to each other is not required. The strange attractor concept seems paradoxical in the conventional frames of control theory. This is a case of transition from CHAOS A to itself.

3.6. Parameter versus added-linear-term control to get regularization

Two control principles merits attention: (1) in linear systems we are more habituated to interaction of variables through some 'amplification factors' (matrix entries); it takes its origin from engineering practice and linear system theories. (2) The second possibility is to influence arbitrary parameters in an arbitrary manner. For example, in linear systems this can define a clear pursuit or equilibrium control separable from time constant control. However, this separation in nonlinear systems can be rarely or cannot be carried out [5].

3.7. Adaptive control

It is a theoretical possibility, if critical parameters leading to or from chaos are adjusted by a trial and error method and in small steps. Only the method of the desired adjustment is specific.

3.8. Control by (minor) changes of operational characteristics

Small changes in characteristics (phase diagram) may lead to significant differences. Such phenomena are well demonstrated by a piecewise linear interval maps (PLMI [11]) as follows: in a multiplex 1D PLMI the motion can be made statistically different: (a) average and variance can be controlled; (b) range of permitted values can be regulated. The system defined around the [0, 1] interval is as follows: L_0: $Y = X + 0.01$ if $X < 0$, L_1: $Y = [1 + (1-a)^2]X = mX$ if $a = 0.1 >= X > 0$, L_2: $Y = [(1-am)/d]X\{ma - [(1-ma)/d]\}$ if $0.1 < X <= a + d$; $L_3 = \{1/[(a+d) - 1]\}X - [1/(a+d)]$ otherwise. If $d = 0$ then no steady state values occur in the middle of [0 , 1] interval and the function is not continuous. If $d > 0$ but small then few values may occur everywhere in [0, 1], the PLMI is continuous. Both systems are chaotic but with different statistical behaviours. At $d = 0$ it is a spike generator–with neurobiological relevence–capable for any strict periodic activities as well; $X = s_n$, $Y = s_{n+1}$ (Fig. 3)

3.9. Transition from CHAOS A to CHAOS B

In bifurcation diagrams, changing a control parameter smoothly, various chaotic behaviours

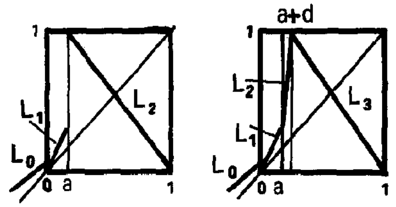

Fig. 3. Next state plot of to very similar PLMI. Corresponding $x_{n+1} = f(x_n)$ are different. In the continuous case the values are distributed in the whole [0,1] interval, in the contiguous one only at in two marginal subintervals. The letters L_i refer to different lines defined in section 3.8.

(a)

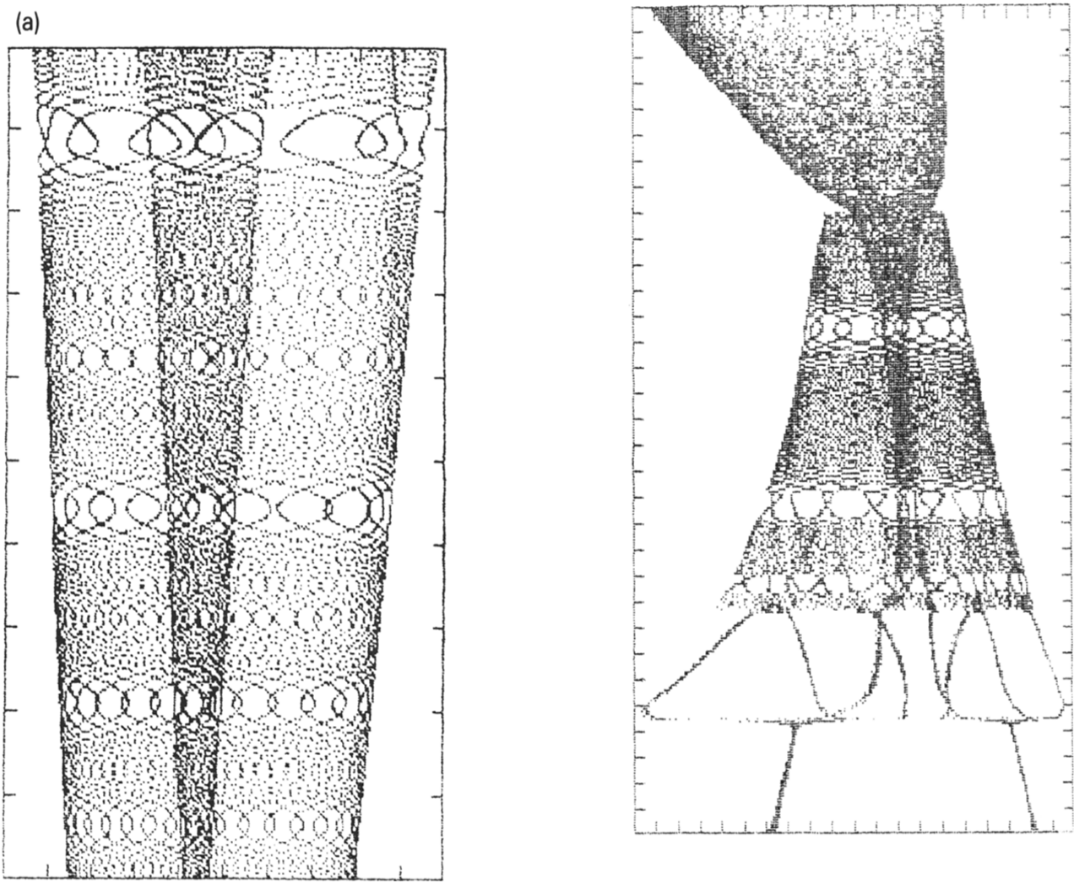

Fig. 4. Bifurcation diagrams displaying dense and sparse ranges of state value sets as the control parameter smoothly changed (section 3.9).

may emerge separated by periodic regimes or even by fixed point. Two chaotic acticities are regarded different if in a bifurcation diagram they are separated by non-chaotic dynamics. Since many bifurcation diagrams are full with such separations, the way from chaos to chaos may occur often. Thus, ranges of a control parameter represent conditions of chaos and the control is the changing this parameter. This is a second way for interchaotic transition, different from that outlined above in section 3.8 (Fig. 4)

(b)

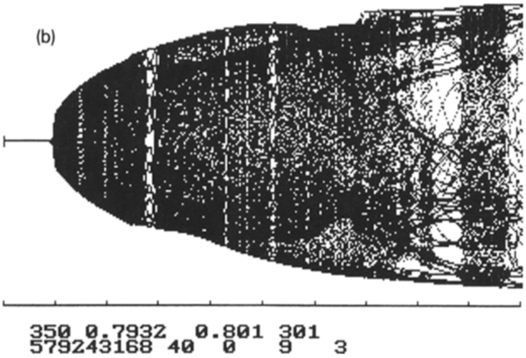

Fig. 4 (continued).

3.10. *Various sorts of parameter controls*

In FNNs, which apply STOs instead of a sharp jump as is done in MCP nets, the slope of usually 'sigmoid' STO is often determined by a single parameter (not mentioning many other occasional parameters). It can be used for control.

Such a parameter control can be carried out in numerous ways. For example replacing the slope constant by a sinusoidal or even by a chaotic time function [12–14].

4. Is the controlled case canonical or not?

The controlled end result cannot be unequivocally determined. All of the regularities (smoother oscillations, constancy or damped, low-amplitude irregularities, more or less regular recurrence, temporal or persistent quasi-synchronous motions of units in nets or even a new chaos, etc.) can be regarded as controlled, demonstrable dynamical problems and systems. That is why *no canonical* definition for regulated non-chaos (freedom from chaos or anti-chaos) exists. Such a definition should include a rather heterogenous group of cases. The problem is pragmatic.

5. Formal, cellular, subcellular or possible molecular implementations

Those parts of neurosciences which should like to interpret structure, behaviour, dynamics and activity of nervous systems by each other seem to be in trouble because of various reasons: (1) the nervous system appears to be highly irregular both in structure and behavior; (2) the so-called subcellular (e.g. ion-channel dynamics) and molecular "organizational level" is not enough known especially as a possible factor in determining activities; (3) the more traditional biochemical world – albeit often can be well studied – does not form a stable bridge between reactions and brain capabilities especially in the more important cases. Here, the chaotic approach is already present; (4) a formal approach can be applied at various 'levels' depending on the meaning attributed to it: the description is syntactical, the interpretation is semantical issue; (5) in the molecular world still there is a rivalization between the traditional statistical approach and only in few cases the concepts of chaos theory are applied; (6) suitable analytical methods – including speedy simulations – are required to get information from the nano–picosecond world and nanometer or finer scale; (7) the reductionist dilemma is present here too: what sorts of macro-phenomena are 'explanaible' by so-called lower organizational levels, if true (at least partially ordered) levels exist at all.

6. Conclusion

The control of chaos is possible in various senses and motivated by various goals:

(1) We emphasize the richness of parameter control.

(2) It should be seen in any particular case what is regarded a '*controlled chaos*'? It is not necessarily the lack of chaos, often it is a forced transition to a better chaos.

(3) Important also to separate autonomous cases from the strictly external control. A sufficiently brutal external control may be dictatorial, frequently leading finally to a motion which is not significantly different from the controller behaviour itself.

(4) It is – as always – good to distinguish the formal aspects from its possible (multiple) meanings. Formal simulations are always neutral and not reflecting necessarily a particular interpretation. This is an additional issue.

(5) Useful considerations for the biochemical or biophysical-molecular 'world' depend on (a) *methodical progress* and (b) probably on the *clarification of the relationships* of chaotic, statistical or (c) indeterministic (Heisenbergian) aspects.

Acknowledgement

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